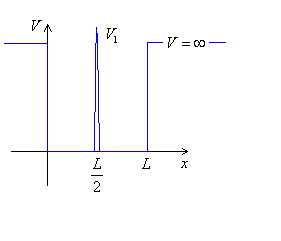
**RSPT Examples**

Let’s take a look at some examples,

**Example**

Suppose we have a square well, with a δ function bump in the middle, V1(x) = V1δ(x-L/2), illustrated below: What is the approximate ground state and ground state energy?



This problem can be solved exactly, but we’ll take the easier route and use RS perturbation theory. So we’ll take the infinite square well w/o the bump to be our unperturbed Hamiltonian, H0, and we’ll take the bump to be our perturbation V1. Then we have:



And our formula for the approximate new ground state is:



Putting this in the |x> basis, our equation for the new ground state wavefunction is:



(recall n = 1 is the ground state). We need to calculate the expectation in the numerator. This is:



Alright, then putting this into our expression for ψ1 we have:



I won’t carry out the sum, but this would be our expression. The approximate new ground state energy is:



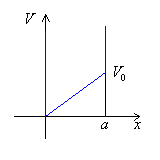
and so this is the approximate new ground state energy to first order.

**Example**

Consider the case where we have V(x) given by the following function:



which looks like,



What are the approximate energy levels of this Hamiltonian? Well our unperturbed states are:



as before, and so the approximate energy levels (to first order) would be:



So the new approximate energy levels are:



**Example**

Let’s redo the infinite square well problem above, but this time we’ll change the boundaries. Let V(x) = ε(x-a) between -a < x < a, and ∞ outside. Then how do the energy eigenvalues change to first order? Well first, the eigenstates are:



Since the unperturbed Hamiltonian commutes with the parity operator, we can write these as:



Then, imposing boundary conditions, we have:\



and,



Normalizing, we have:



So then, the energy corrections will be:



and,



So the energy correction is:



In retrospect, we could see this must be the case, because the unperturbed eigenstates are of definite parity. When we square the eigenstates we get even parity. And even parity function integrated against εx will give zero. So only the -εa term matters. But the wavefunctions are normalized to 1. So we will just get -εa.

**Example. Anharmonic Terms in HO**

Consider a HO with an extra term in the potential (no HO is ideal),



What is the correction to the ground state energy, say, to first order? This would be:



where ω0 = √(k/m). This is most easily evaluated by putting x4 in terms of creation/annihilation operators,



Then we have:



Keeping only terms that will survive,



So the new energy would be:



**Example. Anharmonic Terms in HO**

Let’s do another one,



What is the correction to the wavefunction, say, to first order? This would be:



Let’s specialize to the ground state,



So we need to calculate,



Again, this is most easily evaluated by putting x3 in terms of creation/annihilation operators,



Then we have:



So then, we have:



So there. What about for the first excited state? Then we have:



And we must calculate:



So we come to:



Yeah. Are these states orthogonal to each other? They should be, at least to order γ.



So that’s good.



**Example**

Consider a neutral spin ½ particle in a harmonic well, coupled to an electric field, **E**, through a spin-orbit interaction:



where γ is a small coupling constant. Calculate how the N = 1 energy levels of the system are changed by this Hso perturbation to first order. So the unperturbed energies are:



Or maybe do this in spherical coordinates. Then,



The first order energy level is 3 fold degenerate |n=0,ℓ=1,mℓ = -1>, |n=0,ℓ=1,mℓ = 0>, |n=0,ℓ=1,mℓ = 1>. So to find the first order corrections, we need to diagonalize the Hso in that subspace. So let’s look at the matrix elements,



Let’s consider some symmetries to see which elements are non-zero. Consider parity,



This indicates that each matrix element is actually zero. So the perturbation doensn’t change the energy levels at this order.

**Example**

Consider a neutral spin ½ particle in a harmonic well with a spin-orbit interaction:



where γ is a small coupling constant. What is the ground state of this Hamiltonian? Well, we can write:



So we can write:



The eigenfunctions of the unperturbed H0 are:



These are simultaneous eigenkets of 2, 2. And so they will be simutaneous eigenkets of the perturbation too. The energies/states will be:



The ground state is characterized by n = 0, ℓ = 0, s = ½, j = 0, mj = 0. And its energy will be:



and will be non-degenerate. The first excited state energies will be characterized by n = 0, ℓ = 1. These will be characterized by the quantum numbers:



which simplifies somewhat to:



The first excited state will be when j = ½ and will be 2-fold degenerate. So we’ll have:



The second excited state will be where j = 3/2, and will be 4-fold degenerate:



So the original 6-fold degeneracy is split by the SO interaction. We can reproduce these results with PT. Our first order state is degenerate. We have to diagonalize the 1st excited state(s) degenerate subspace spanned by:



Way back in the S-algebraic file we found the matrix representation of the angular momentum operator for s = 1 (same as ℓ = 1) and s = ½. These are:



So our spin-orbit perturbation looks like this:



To diagonalize this I guess we need to put the matrices together into a 6-dimensional space:



The eigenvalues of this matrix follow from setting the determinant to zero. The determinant is just the product of the block determinants.



Well the roots are:



So altogether, I’m getting λ = -2√2 is 2-fold degenerate, and √2 is 4-fold degenerate. So the perturbed energies would be:



This matches what we found above.